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PHYLLOTAXIS OF CONES.

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In the summer of 1870 I examined a large number of cones of several species of Coniferæ to see if there was any variation in their leaf arrangement. It has long been well known that the scales or leaves of cones show very plainly a certain number of parallel spiral whorls twisting to the right and a different number twisting to the left. A closer examination will also usually reveal other parallel whorls (one or more in each direction) with numbers differing from those most easily seen. By beginning with the simplest forms of alternate leaf-arrangement, as the elm ($\frac{1}{2}$), and sedges ($\frac{1}{3}$); and then to the more common but more complicated, as the cherry ($\frac{2}{5}$), and American larch ($\frac{3}{8}$), it is found that in these fractions the numerator expresses the number of times we pass around the stem to find a leaf directly over the one with which we started, while the denominator indicates the number of vertical ranks or rows of leaves up and down the stem. This is nicely proven to be true in the case of a fraction with large numerator and denominator in the leaves of *Yucca filamentosa*, where the fraction is thirteen thirty-fourths, if memory is not at fault. In Yucca the bases of the leaves are so broad that they reach about half-way around the stem, so it is easy to see which is below or outside of all the others. The fractions above mentioned also express the angular divergence or show the proportion of the

whole circumference which intervenes between any two consecutive leaves of the same spiral whorl. Stretch a wire or band with marks or appendages so as to be alternate, two-ranked as are the leaves in the elm; then by giving the band a twist, it brings the marks three-ranked, like the sedges; still farther torsion brings them five-ranked, like the leaves of a cherry tree; still more twist and they stand as the scales of the American larch, which is expressed by the fraction three-eighths.

The most common series of fractions found in alternate leaves is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{5}{13}$, $\frac{8}{21}$, $\frac{13}{34}$, $\frac{21}{55}$, $\frac{34}{89}$, etc. The relations of these several numerators and denominators have been repeatedly shown by various authors.

After the first two fractions, each succeeding one may be made by adding both of the previous numerators for its numerator and both of the previous denominators for its denominator. Each denominator is the same as the second succeeding numerator. "Also, taking the orders of secondary spirals nearest the vertical line, on each side, right and left, the number of parallel spirals of the lower order of these two will give the numerator; and this number, added to the number of parallel spirals of the higher order will give the denominator."—*Henfrey*. Also "the number of the parallel secondary spirals is the same as the common difference of the numbers on the leaves that compose them."—*Gray*. These relations enable us to number easily each scale of any cone, or count the spirals each way, and then determine with accuracy the fraction expressing its Phyllotaxis. Balfour and Gray in their text books say the Phyllotaxis is uniform in the same species, and that one direction or the other generally prevails in each species, and that both directions are sometimes met with in different cones of the same tree. Several other text books make the same assertions. Most authors on this subject with which I am familiar say there are only rare cases of other series of spirals. P. Duchartre mentions two other series, viz: $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{7}$, $\frac{3}{11}$, $\frac{5}{18}$, $\frac{8}{29}$, etc., $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{9}$, $\frac{3}{14}$, $\frac{5}{23}$, etc., and observes that the same relation exists in different fractions of each series as exist in the fractions of the more common series.

Mr. Hubert Airy recently read a paper before the Royal Society, England, an abstract of which is given in "Nature" for March 6, 1873. After mentioning some experiments which show the intimate relations of different fractions of the common series,

he adds: "It also appears that the necessary sequence of these successive steps of condensation, thus determined by the geometry of the case, does necessarily exclude the non-existent orders $\frac{1}{4}$, $\frac{3}{7}$, $\frac{4}{9}$, $\frac{4}{11}$, etc." This conclusion "determined by the geometry of the case," proves to be only an incorrect theory, as shown by the following:

I examined nearly all the cones (one hundred and fifty-five) which grew upon a Norway spruce, seventy-four of which showed five parallel spirals to the right and eight to the left; while seventy-four showed eight spirals to the right and five to the left. Five cones had seven spirals to the right and four spirals to the left. One cone had four spirals to the right and six to the left, and one cone had six spirals to the right and four to the left. I will try to tabulate this and others in a briefer manner:—

NORWAY SPRUCE. . . .	74	Cones	had	5	spirals	to the	right,	8	to the	left.
On Tree No. 1.	74	"	"	8	"	"	"	5	"	"
	5	"	"	7	"	"	"	4	"	"
	1	"	"	4	"	"	"	6	"	"
	1	"	"	6	"	"	"	4	"	"
On Tree No. 2.	18	"	"	5	"	"	"	8	"	"
	21	"	"	8	"	"	"	5	"	"
	1	"	"	4	"	"	"	6	"	"
On Tree No. 3.	5	"	"	5	"	"	"	8	"	"
	19	"	"	8	"	"	"	5	"	"
	1	"	"	7	"	"	"	4	"	"
On Tree No. 4.	23	"	"	8	"	"	"	5	"	"
	17	"	"	5	"	"	"	8	"	"
	4	"	"	7	"	"	"	4	"	"
	2	"	"	4	"	"	"	7	"	"
	1	"	"	4	"	"	"	6	"	"
	1	"	"	6	"	"	"	4	"	"
On Tree No. 5.	34	"	"	5	"	"	"	8	"	"
	44	"	"	8	"	"	"	5	"	"
	6	"	"	4	"	"	"	7	"	"
	3	"	"	7	"	"	"	4	"	"
	1	"	"	4	"	"	"	6	"	"
	2	"	"	6	"	"	"	4	"	"
On Tree No. 6.	63	"	"	5	"	"	"	8	"	"
	53	"	"	8	"	"	"	5	"	"
	1	"	"	4	"	"	"	7	"	"
	6	"	"	7	"	"	"	4	"	"
	4	"	"	4	"	"	"	6	"	"
	8	"	"	6	"	"	"	4	"	"
On Tree No. 7.	9	"	"	8	"	"	"	5	"	"
	13	"	"	5	"	"	"	8	"	"
PINUS PUMILIS.	10	"	"	5	"	"	"	8	"	"
	9	"	"	8	"	"	"	5	"	"
EUROPEAN LARCH. . .	29	"	"	5	"	"	"	13	"	"
	51	"	"	13	"	"	"	5	"	"
	3	"	"	4	"	"	"	7	"	"
	2	"	"	7	"	"	"	4	"	"
	1	"	"	3	"	"	"	6	"	"

BLACK SPRUCE.	80	Cones	had	5	spirals	to the	right,	8 to	the left.
Cones for 1869.	65	"	"	8	"	"	"	5 "	"
	3	"	"	4	"	"	"	7 "	"
	2	"	"	7	"	"	"	4 "	"
	1	"	"	4	"	"	"	6 "	"
	3	"	"	6	"	"	"	4 "	"
Same tree in 1870. . . .	26	"	"	5	"	"	"	8 "	"
	23	"	"	8	"	"	"	5 "	"
	2	"	"	7	"	"	"	4 "	"
AMERICAN LARCH. . .	30	"	"	3	"	"	"	5 "	"
	34	"	"	5	"	"	"	3 "	"
	3	"	"	4	"	"	"	6 "	"
	1	"	"	6	"	"	"	4 "	"
	1	"	"	7	"	"	"	4 "	"
	1	"	"	3	"	"	"	4 & 10 "	"

In all of these cases it was possible to see other spirals, but I have mentioned those most apparent in each case. For instance, in the most common forms of Norway spruce, there were spirals with three rows, eight and twenty-one one way, and five and thirteen the other way. Other cones showed three and seven one way, and four and eleven the other.

To cut this article short, the fractions for most cones of Norway spruce was $\frac{13}{34}$, for others it appears to be $\frac{11}{29}$, and for others $\frac{10}{26}$. By operating with the fraction $\frac{11}{29}$ and other numbers of spirals on the cones in the same way as we may on the most common forms, we get this series of fractions, viz: $\frac{1}{4}$, $\frac{3}{7}$, $\frac{4}{11}$, $\frac{7}{18}$, $\frac{11}{29}$, etc. Other cones noticed in the table as having four and six spirals, had also two, ten, and sixteen. The fraction for these was $\frac{10}{26}$, and would be found in a series $\frac{2}{4}$, $\frac{2}{6}$, $\frac{4}{10}$, $\frac{6}{16}$, $\frac{10}{26}$, $\frac{16}{42}$, etc. The latter we observe, when each fraction is reduced to its lowest terms, is the same as the first or most common fractions mentioned. Most cones of the European larch had the phyllotaxis expressed by the fraction $\frac{8}{21}$, others by $\frac{7}{18}$, one other by $\frac{6}{15}$. This latter cone had three, six and nine spirals, and falls into the following series, viz: $\frac{3}{6}$, $\frac{3}{9}$, $\frac{6}{15}$, $\frac{9}{24}$, $\frac{15}{39}$, etc. Most cones of the American larch fall under the fraction $\frac{3}{8}$, others $\frac{4}{10}$, others $\frac{3}{7}$.

In these few examples the same number of parallel spirals is about equally divided in the two directions, right and left. They also show that other series than the one usually accepted as almost universal, are not uncommon, as they may be found on a variety of coniferous trees, though in smaller numbers.

Plants with the leaves opposite generally have them four-ranked up and down the stem, and then the leaves are said to *decussate*. If we start with a plastic stem of this nature we get a fraction $\frac{2}{4}$ to express it; giving the axis a slight twist we get $\frac{2}{6}$, another

twist we get $\frac{4}{10}$, another twist $\frac{6}{16}$, etc. Some of our cones, then, fall into the phyllotaxis of opposite leaves the same as though the stem were more or less twisted. The single cone of the European larch which indicated the fraction $\frac{6}{15}$ (a fraction requiring a division of numerator and denominator by three to reduce it to its lowest terms) falls under decussate whorls of three for its simplest fraction.

I leave any further consideration of this matter showing the relations of the fractions to each other, etc., to those who have a greater skill in mathematics than myself. My examples indicate that we may look for some curious series of fractions by diligently examining the phyllotaxis of a great number of plants of many different species.

It would be interesting to know whether there are any cones which fall into series beginning with decussate whorls of four or more scales.

ON THE DISTRIBUTION OF CALIFORNIAN MOTHS.*

BY A. S. PACKARD, JR.

THE Phalænidæ (Geometrids) of California (including Oregon and Nevada) seem to be composed of four elements: (1) of species of genera exclusively American (North and South). Such are *Chœrodes*, *Sicya*, *Hesperumia*, *Tetracis*, *Azelina*, *Gorytodes* and *Metanema*. Certain species of these, with several of *Tephrosia* (a genus largely found in the New World) are the most characteristic of the Pacific slope of the United States.

(2) The species next most characteristic belong to the following genera: — *Halia*, *Tephrina*, *Selidosema* and *Heterolocha*. Species of these groups occur in Europe, but especially (all except *Halia* which has a species (*H. novaria*) living in northern Europe) in southern Europe, around the Mediterranean Sea, western Asia, and Asia Minor; while species of Heterolocha occur in Abyssinia and South America (Quito).

(3) The next group comprises a few arctic or circumpolar species of *Coremia*, *Cidaria* and *Larentia*, or of cosmopolite genera

* Extracted from a communication presented to the Boston Society of Natural History, May 7, 1873.